S.No. 8043 **24DPMAE02**

(For the candidates admitted from 2024-25 onwards)

M.Sc. DEGREE EXAMINATION, AUGUST 2025.

First Semester

Maths

GRAPH THEORY AND APPLICATIONS

Time : Three hours Maximum : 75 marks

PART A — ($10 \times 2 = 20$ marks)

Answer ALL questions

1. Define a cycle.
2. Define a directed graph.
3. Define connectivity of a graph
4. Define a tree.
5. Define an independent set.
6. Define a Hamilton path of a graph.
7. Define chromatic number.
8. What are Class 1 graphs?
9. Define planar graph.
10. Define dual of a graph.

PART B — ($3 \times 5 = 15$ marks)

Answer any THREE questions.

11. Prove that if a simple graph G is not connected, then G^C is connected.
12. Prove that if e is not a loop of G, then $\tau(G) = \tau(G-e) + \tau(G \cdot e)$.
13. Let G be a simple graph with $n \geq 3$ vertices. If for every pair of nonadjacent vertices u, v of G, $d(u) + d(v) \geq n$, then show that G is Hamiltonian.
14. Prove that if G is k-critical, then $\delta \geq k-1$.
15. Show that K_5 is nonplanar.

PART C — ($5 \times 8 = 40$ marks)

Answer ALL questions

16. (a) Show that a graph is bipartite if and only if, it contains no odd cycles.

Or

(b) Stare and prove Moon's Theorem.

17. (a) Show that for any loopless connected graph $G, \kappa(G) \leq \lambda(G) \leq \delta(G)$.

Or

(b) State and prove Jordan's Theorem.

18. (a) State and prove Hall's Theorem.

Or

(b) State and prove Chvatal and Erdös Theorem.

19. (a) Prove that if G is a loopless bipartite graph, $\chi'(G) = \Delta(G)$.

Or

(b) State and prove Vizing-Gupta's Theorem.

20. (a) Show that for a Connected plane graph G, $n - m + f = 2$, where f denotes the number of faces of G.

Or

(b) State and prove Heawood five-color theorem.

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